Radiative Processes in Planetary Atmospheres — Homework 1

Due: September 17, 2003

Problem 1.1 (30 points)

A black land surface with a temperature of 15 ^oC emits radiation at all frequencies.

a) *What would be the emitted radiances at 0.5 μm, 1200 cm^{-1}, and 35 GHz? Use appropriate Plank functions in the calculations.*

Solution for Problem 1.1 (a).

Use the following definition of the Plank function (RT ATOC5560 lecture 3) to calculate the answer to the above problem.

- **Plank function** can be expressed in wavelength, frequency, or wavenumber domains as

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5(\exp(hc/k_B T\lambda) - 1)} \qquad [3.1]$$

$$B_{\tilde{\nu}}(T) = \frac{2h\tilde{\nu}^3}{c^2(\exp(h\tilde{\nu}/k_B T) - 1)} \qquad [3.2]$$

$$B_\nu(T) = \frac{2h\nu^3 c^2}{\exp(h\nu c/k_B T) - 1} \qquad [3.3]$$

where $\boldsymbol{\lambda}$ is the wavelength; $\tilde{\nu}$ is the frequency; $\boldsymbol{\nu}$ is the wavenumber; $\boldsymbol{h}$ is the Plank's constant; $\boldsymbol{k_B}$ is the Boltzmann's constant ($k_B = 1.3806 \times 10^{-23}$ J K^{-1}); $\boldsymbol{c}$ is the velocity of light; and $\boldsymbol{T}$ is the absolute temperature of a blackbody.

The answers are:

Eq. [3.1]: 1.6×10^{-28} J sec^{-1} m^{-3} sr^{-1} = 1.6×10^{-34} J sec^{-1} m^{-2} sr^{-1} / μm

Eq. [3.3]: 5.2×10^{-4} J sec^{-1} m^{-1} sr^{-1} = 5.2×10^{-2} J sec^{-1} m^{-2} sr^{-1} / (cm^{-1})

Eq. [3.2]: 1.1 x 10 $^{-16}$ J m $^{-2}$ sr $^{-1}$ = 1.1 x 10 $^{-16}$ J sec $^{-1}$ m $^{-2}$ sr $^{-1}$ / Hz

b) *What is the wavelength of the maximum intensity of the black body at 7000K temperature (use Wien's Displacement law)?*

Solution for problem 1.1 (b)
The Wien's displacement law states that the wavelength of the maximum intensity of black body radiation is inversely proportional to the temperature:
λ=a/T,
where **a** is 2.897 x 10 $^{-3}$ m deg.
After substituting T=7000 into the equation, we find that the maximum intensity is reached at 414 nm or 0.414 μm or 4.14 x 10^{-7} m.

Problem 1.2 (30 points)
a) Ozone is a very important climatological parameter of the Earth atmosphere (see http://www.epa.gov/docs/ozone/science/index.html). Ozone amount is measured in Dobson units. A Dobson unit is a milli-atmospheric-cm, where an atm-cm is the thickness of the gas if reduced to standard temperature (273 K) and pressure (101.3 kPa). Total Ozone Column measured by Earth Probe TOMS over Antarctica on September 15 in 2001 during the so-called "Ozone Hole" event (at -71.00 Degrees Latitude -6.00 Degrees Longitude) is only 148 DU.
http://toms.gsfc.nasa.gov/ozone/ozone.html
Convert the total ozone in Dobson units to absorber amount in g/cm^2. Hint: a gas at standard temperature and pressure occupies 2.241 x 10^4 cm^3/mol.

Solution for problem 1.2 (a):

$$u = \Delta z \times \rho = \Delta z \times \frac{M}{V} = \frac{(0.148cm)(48g/mol)}{2.241 \times 10^4 cm^3/mol} = 3.17 \times 10^{-4} g/cm^2$$

b) A radiosonde observation in Denver, Colorado on 16 February 2001 showed as much as 2.00 g/cm2 water vapor amount while the forecast was only for 0.15 g/cm2 of water vapor. Unpredicted dense fog caused the diversion of flights from Denver International airport for 18 hours. Usually water vapor is measured in precipitable cm or inches, which is the height of the column of water resulting from condensing all of the water vapor out. Convert 0.15 and 2.00 g/cm2 of water vapor to precipitable inches.

Solution for problem 1.2 (b):
Use definition of optical path:
$u = \int \rho dz$,
where ρ is constant and for water is 1g/cm3. Thus, u=ρΔz. Then, use 1 in=2.54 cm to convert to inches:

$$\Delta z = \frac{(0.15 g/cm^2)}{(1.00 g/cm^3)(2.54 cm/in)} = 0.06 in \qquad \Delta z = \frac{(2.00 g/cm^2)}{(1.00 g/cm^3)(2.54 cm/in)} = 0.8 in$$

c) Carbon dioxide concentration is measured in parts per million by volume. There are small seasonal and geographic variations and an increasing trend. The maximum value in Texas as measured by the Climate Monitoring and Diagnostic Laboratory is about 390 ppmv (see http://www.cmdl.noaa.gov/ccgg/towers/diurnal.html). Convert this concentration to absorber amount in g/cm2 above Texas (457 m altitude, 990 mb pressure) and above the Mauna Loa observatory (3400 m altitude, 680 mb pressure).

Solution for problem 1.2 (c):
Use hydrostatic equation to find density of air:

$$\frac{dp}{dz} = -\rho_{air} g$$

After re-arranging equation and taking integral of both sides of equation, we get:

$$\int \rho_{air} dz = -\frac{1}{g}\int_{ps}^{0} dp = \frac{p_s}{g}$$

Use definition of optical path, therefore for air optical path:

$$u_{air} = \int \rho_{air} dz$$

To calculate optical path for CO2 use mixing ratio:

$$u_{co2} = q_{air} u_{air}$$

Do not forget to convert volume-mixing ratio to mass-mixing ratio using molecular mass of CO2 (44 g/mole) and air (29 g/mol):

$$q_a = \frac{m_{co2}}{m_{air}} q_v$$

Therefore,

$$u = \frac{q_a}{g} p_S = \frac{(3.90\times10^{-4})(44.0/29.0)(9.90\times10^{4}\,N/m^2)}{(9.8m/s^2)} = 5.98kg/m^2 = 0.598g/cm^2$$

$$u = \frac{q_a}{g} p_S = \frac{(3.90\times10^{-4})(44.0/29.0)(6.80\times10^{4}\,N/m^2)}{(9.8m/s^2)} = 4.11kg/m^2 = 0.411g/cm^2$$

Problem 1.3 (40 points).

Measurements by a narrow field of view tracking sun-photometer were done at Arosa, Switzerland (station is located at 0.8 atm pressure level). One morning the instrument records the following measurements at 0.38 μm wavelength as Sun rises above the horizon:

SZA (degrees)	*V records from the instrument (W/m2/mV)*
50	*616*
40	*760*
30	*873*
20	*1286*

a) *What is the aerosol optical depth at 0.38 μm? Use Lambert-Beer law. Note, there is another source of attenuation at 0.38 μm wavelength besides aerosols, which is Rayleigh scattering by atmosphere. Rayleigh extinction coefficient at 380 nm is about 0.425 atm^{-1} at sea level pressure. If the instrument would be placed outside of the atmosphere it would measure 2275.6 W/m2/mV.*

Solution for problem 1.3(a)

Use the following form of Beer-Bouguer-Lambert Law to calculate extinction of atmosphere from your measurements:

$$\ln V = \ln V_0 - \mu\,(\tau_{Ray} + \tau_{aer}) = \ln V_0 - \mu\beta P - \mu\tau_{aer}, \quad (100)$$

where V_0 is a measure of the spectral irradiance at the top of atmosphere, $\mu=1/\cos\theta$, θ is a measurement angle, P is an atmospheric pressure at the measurement location (0.8 atm), β is Rayleigh extinction coefficient.

Thus,

$\tau_{aer}=\ln (V_0/V)/ \mu- \beta P$

SZA	Optical depth
50	0.5
40	0.5
30	0.5
20	0.2

b) *Did you need V_0 to solve for aerosol optical depth? Can you use another method? What assumptions about the atmosphere did you have to make to derive aerosol optical depth?*

Solution for problem 1.3(b)

The Langley plot technique can be used to find atmospheric optical depth between instrument and Sun. The Langley method uses the plot of log of sun-photometer measurement as function of air-mass. The Eq. (100) can be thought as liner function Y=a+bX, where X is the air-mass (μ), y is the log of the measurement, a is the intercept of the line with y-axes or the log of the measurement at the top of atmosphere ($\ln(V_0)$), and b is the negative of total optical depth.

However, the Langley plot technique assumes that the atmosphere is homogeneous (uniform) and is not changing during the time it takes to do all measurements. This assumption will be required so we can use slope as a measure of optical depth. If the optical depth changes from measurement to measurement (from one SZA to another, see the last measurement in table) then the slope in the Langley plot is not constant and can not be approximated as a straight line. After verifying that the plot is a straight line (use first 3 measurements from the table), the slope can be found from two points,

$$\tau = -\frac{\ln V_1 - \ln V_2}{m_1 - m_2} = -\frac{\ln 616 - \ln 760}{1.55 - 1.31} = 0.875$$

Since radiation at 380 nm is attenuated by aerosols and Rayleigh scattering, to find aerosol optical depth we need to subtract Rayleigh optical depth (0.425*0.8=0.34) from total optical depth.
Thus, aerosol optical depth is 0.87-0.34=0.53, which is slightly different from original measurements and is affected by rounding errors of measurement.
The Langley plot method also requires that the measurements are taken in the narrow wavelength range that the Beer-Bouguer-Lambert law can be used to calculate transmission.